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Original Article

Closed form solution to seal performance of power law fluid with a seal ring and a rotating seal plate

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Abstract

This paper deals with seal performance of a non-Newtonian power-law fluid. The mechanism comprises a primary seal ring flexibly mounted on a support and a secondary seal plate capable of rotating about a shaft. Solution to the seal performance is obtained unlike earlier authors in closed form subject to a set of boundary conditions compatible with the fluid motion and as such the leakage velocity- components have been determined analytically for pseudo plastic fluids and dilatants. Finally numerical examples have been illustrated in case of some non-Newtonian fluids.

1. Introduction

The seal [1] consists of a fixed plate called primary seal in form of a ring mounted on a support and a circular plate called secondary seal placed parallel to hand over the seal ring as illustrated in Fig1.The secondary seal is capable of rotating about a shaft which acts as a common axis for both the seals. The small passage/gap between the primary and secondary seals is called film thickness. The power-law fluid is allowed to enter the region between the secondary seal (plate) and the annular part of the ring through the latter; simultaneously the secondary seal is set to rotate about the shaft, bringing about vigorous motion of the fluid in the seal region. However, because of radial flow of the fluid there occurs from the seal region a flux of the fluid called leakage. Needless to mention that the main purpose of the seal is to minimize the leakage. Prawal Sinha[1] and his coauthor mostly gave numerical solution to non-Newtonian power-law fluid flow through the seal .In this paper is worked out an analytical solution to the fluid flow effecting some minor changes in the boundary conditions but without sacrifice of the physical significance and generality.

2. Equations of motion for the seal

Let (v_r, v_θ, v_z) be the components of the fluid velocity *in* r, θ, z directions at (r, θ, z) *point* in cylindrical coordinates with respect to the origin at the centre of the ring. Then the pressure equation[1] is

$$\frac{dp}{dr} = \frac{\partial_{zr}}{\partial z} + \rho\frac{v_\theta^2}{r} \tag{1}$$

$$\frac{\partial\tau_{z\theta}}{\partial z} = 0 \tag{2}$$

where p is the pressure, a function of the radial distance r;τ_{zr} ,$\tau_{z\theta}$ *are stresses and* ρ *the fluid density.*

The equation of continuity is

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \quad for\ 0 \le r < r_1 \tag{3}$$

$$r_1 v_{r_1} = rv_r = r_2 v_{r_2} \quad \text{for } r_1 \le r \le r_2 \tag{3.1}$$

The shear stresses[1] for the power- law model[1] are

$$\tau_{zr} = m\left|\frac{\partial v_r}{\partial z}\right|^{n-1}\frac{\partial v_r}{\partial z} \tag{4}$$

$$\tau_{z\theta} = m\left|\frac{\partial v_\theta}{\partial z}\right|^{n-1}\frac{\partial v_\theta}{\partial z} \tag{5}$$

where m and n are constants called consistency index and flow index respectively for the fluid flow.

The boundary conditions[1] are

$$v_\theta = o\ at\ z = 0\ and\ v_\theta = r\omega\ \ at\ z=h \tag{6}$$

where h is the thickness of the film,

andω *the constant angular velocity of the* seal about the shaft.

By use of equation (2), (5) and (6) sinha[1] and his coauthor obtained

$$v_\theta = \frac{r\omega z}{h} \tag{7}$$

and using (7) and (4) in equation (2) they[1] obtained

$$\frac{dp}{dr} = mn\frac{\partial^2 v_r}{\partial z^2}\left|\frac{\partial v_r}{\partial z}\right|^{n-1} + \frac{\rho r\omega^2 z^2}{h^2} \tag{8}$$

Now unlike Sinha[1] and his coauthor in this paper an attempt is made to look for closed- form solution to equation(8) and (3) i.e. to find explicitly the pressure p and the velocity components , v_z, v_r subject to the boundary conditions relevant to the seal performance.

Analytical Solution in closed form

Since the radial velocity component

v_r *of the fluid is a function of both* r and z coordinates, we can have the non-dimensional velocity component as

$$\bar{v_r} = R(r)Z(z) \tag{9}$$

where

$R(r)$ *and*$Z(z)$ *are respectively* functions of r and z, putting which in (8) and introducing non- dimensional coordinates

$$\bar{r}=\frac{r}{r_2},\bar{r_1}=\frac{r_1}{r_2}, \bar{z}=\frac{z}{h}, \ \bar{v_r}=\frac{v_r}{r_2\omega}, \quad \bar{p}=\frac{p-P}{P_{i-P}} \tag{10}$$

we get

$$\frac{1}{\bar{E_u}}\frac{d\bar{p}}{d\bar{r}}=\frac{1}{\mathrm{R_n}}\frac{\partial^2\bar{v_r}}{\partial\bar{z}^2}\left|\frac{\partial\bar{v_r}}{\partial\bar{z}}\right|^{n-1} + \bar{r}\,\bar{z}^2 \tag{11}$$

where $$\bar{E}_u=\omega r_2/\sqrt{\frac{(P_i-P)}{\rho}}, \frac{1}{\mathrm{R_n}} = \frac{\mathrm{R_e}}{(\mathrm{R_{e\omega}})^2}=\frac{mn}{\omega h^2\rho}\left(\frac{\omega r_2}{h}\right)^{n-1} \tag{12}$$

r_1, r_2 are the inner and outer radii of the ring so that r_2 is the radius of the plate while P_i is the inlet pressure and P the ambient pressure;

$\bar{Q}=\frac{Q}{h(\omega r_2)r_2}$ where Q is the flux and

$$\mathrm{R_{e\omega}} = \frac{h(\omega r_2)\rho}{mn}\left(\frac{h}{\omega r_2}\right)^{n-1} \tag{13}$$

$$\mathrm{R_e} = \rho\frac{\omega r_2^2}{\mathrm{mn}}\left(\frac{h}{\omega r_2}\right)^{n-1}$$

Employing (9) in (11) we after dropping the bar sign can get the following equation

$$\frac{1}{E_u^2} = \frac{1}{r}\frac{dp}{dr} = -\frac{R^n(r)}{R_n r}\frac{\mathrm{d^2Z}}{\mathrm{dz^2}}\left|\left(\frac{dZ}{dz}\right)\right|^{n-1} + \mathrm{z}^2 = -\mathrm{c(constant)} \tag{14}$$

$0 \le r \le r_1$ (The bar sign is kept understood) so that

$$\frac{1}{E_u^2}\frac{dp}{dr} = -cr \tag{15}$$

With the boundary conditions

$$\mathrm{p}=p_i=1 \text{at r}=r_1 \text{ and p=0 at r=1} \tag{16}$$

Solution to (15) subject to (16) yields the pressure equation

$$\mathrm{P}=1-\mathrm{c}\frac{E_u^2(r^2-r_1^2)}{2} \tag{17}$$

$$\text{where c}=\frac{2}{E_u^2(1-r_1^2)} \tag{17.1}$$

so that $\mathrm{p}=\frac{1-r^2}{1-r_1^2}$

To obtain equation (14) amenable to analytical solution, of course, with fulfillment of some other boundary conditions we must have

$$\frac{R^n}{R_n r}=c_{1(constant\ )} \quad (18)$$

which because of (9) gives

$$v_r(r,z)=(c_{1R_n}r)^{1/n}Z(z) \quad (19)$$

for $0\leq r\leq r_1, z\neq 0$

and because of the equation of continuity suggesting the rate of flow into the annular region equal to the flux,

$$v_r(r,z)=\frac{r_1}{r}(c_{1R_n}r_1)^{1/n}Z(z) \text{ for } r_1\leq r\leq r_2=1, z\neq 0 \quad (20)$$

so that equation(14) leads to

$$c_1\frac{d^2Z}{dz^2}|(\frac{dZ}{dz})|^{n-1}=(c+z^2)$$

which in tandem with (19) subject to the boundary conditions

$\frac{\partial v_r}{\partial z}=0$ at z=1

$v_r=0$ at z=1 (21)

z(1) =0

can be solved as

$$\frac{dZ}{dz}=-[\frac{n}{c_1}\{c(1-z)+(\frac{1-z^3}{3})]^{1/n} \quad (22)$$

$$Z=\int_z^1[\frac{n}{c_1}\{c(1-z)+(\frac{1-z^3}{3})]^{1/n}dz \quad (23)$$

and as such in view of relations (9) and (18)

$$v_r(r,z)=(R_n rn)^{\frac{1}{n}}\int_z^1\left[\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}\right]^{\frac{1}{n}}dz \quad (24)$$

$0\leq r\leq r_1, z\neq 0$

Now because of (19), equation (3) turns out to be

$$\frac{\partial v_z}{\partial z}=-(c_{1R_n})^{1/n}r^{\frac{1-n}{n}}Z.(\frac{1+n}{n}) \quad (25)$$

Boundary conditions for velocity component $v_z(r,z)$ are

$v_z(r,1)=0$ (26)

$v_z(r,0)=v_1 r^{\frac{1-n}{n}}$ $for\ 0\leq r<r_1,$ (27)

$v_z(r,0)=0$ $for\ 0\leq r\leq r_2,$ (28)

which hold on integration of (25) by use of (23)

$$v_z(r,z)=-(c_{1R_n})^{1/n}r^{\frac{1-n}{n}}(\frac{1+n}{n})\int_1^z Z(t)dt \quad (29)$$

(Integration by parts)

$$=-(c_{1R_n})^{1/n}r^{\frac{1-n}{n}}\frac{1+n}{n}[zZ(z)-Z(1)-\int_1^z\frac{dZ}{dt}tdt]$$

$$v_z(r,z)==-(nR_n)^{1/n}r^{\frac{1-n}{n}}\frac{1+n}{n}\int_z^1\{c(1-t)+\frac{1-t^3}{3}\}^{1/n}(z\text{-}t)dt \quad z\neq 0 \quad (30)$$

Further equation (24) suggests the boundary conditions for $v_r(r,z)$ as

$v_r(r,0)=v_0 r^{1/n}$ $for\ 0\leq r\leq r_1,$ (31)

$v_r(r,0)=0$ $for\ 0\leq r\leq r_2,$ (32)

$$\text{where}\quad v_0=(R_n n)^{\frac{1}{n}}\int_0^1\left[\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}\right]^{\frac{1}{n}}dz\geq 0 \quad (33)$$

$$v_1=(R_n n)^{\frac{1}{n}}\frac{1+n}{n}\int_0^1\left[\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}\right]^{\frac{1}{n}}zdz\geq 0 \quad (34)$$

In the light of (30).

3. Performance of a newtonian fluid

For a Newtonian fluid n=1 and then in view of equations (24) to (34) the velocity components become

$$v_r(r,z)=(R_n r)\left(\int_z^1[\{c(1-z)+\left(\frac{1-z^3}{3}\right)\}]^1dz \text{ for } r\in[0,r_1), z\neq 0\right.$$

$$=R_n r\{\left(\frac{1}{3}+c\right)(1-z)-c(\frac{1-z^2}{2})-\frac{1-z^4}{12}\} \quad (35)$$

Because of equation (20) one gets using (23)

$$v_r(r,z)=\frac{r_1^2}{r}(R_n)\left(\int_z^1[\{c(1-z)+\left(\frac{1-z^3}{3}\right)\}]^1dz, \text{ for } z\neq 0, r\in[r_1,r_2] \quad (36\right.$$

$$=\frac{r_1^2}{r}(R_n)[\left(c+\frac{1}{3}\right)(1-z)-c\frac{1-z^2}{2}-\frac{1-z^4}{12}] \quad (36)$$

$$v_r(r,0)==R_n r(\frac{c}{2}+\frac{1}{4}) \qquad r\in[0,r_1] \quad (37)$$

Using properties of definite integrals in (30), we get

$$v_z(r,z)=2R_n[\int_z^1\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}zdz-z\int_z^1\{c(1-t)+\frac{1-t^3}{3}\}dt]$$

$$=2R_n[\left\{\frac{(c+\frac{1}{3})(1-z^2)}{2}-\frac{c(1-z^3)}{3}-\frac{1-z^5}{15}\right\}-z\{\left(\frac{1}{3}+c\right)(1-z)-c(\frac{1-z^2}{2})-\frac{1-z^4}{12}\}] \quad (38)$$

$$v_z(r,0)=R_n(\frac{c}{3}+\frac{1}{5}) \qquad for\ 0\leq r<r_1, \quad (39)$$

Equation(38) can be further simplified as

$$v_z(r,z)=R_n[(c+\frac{1}{3})(1-z)^2+\frac{c(1-z^3)}{3}+\frac{1-z^5}{30}] \quad z\neq 0$$

4. Performance of non-newtonian fluid

For non-Newtonian fluid the velocity components $v_r(r,z)$ and $v_z(r,z)$ as in (24) and (31) can be formulated by Binomial expansion in series as

$$v_r(r,z)=\left((R_n rn)^{\frac{1}{n}}\left(c+\frac{1}{3}\right)^{\frac{1}{n}}\int_z^1\{1-\frac{cz+\frac{z^3}{3}}{(c+\frac{1}{3})}\right\}dz \qquad 0\leq z\leq 1$$

SInce $z\leq 1, \frac{cz+\frac{z^3}{3}}{(c+\frac{1}{3})}\leq 1;$ *considering* $c>>1$ depending on (12) and (17.1) we can replace z^3 *by* z in the above integral for desired approximation so that

$$v_r(r,z)=(R_n rn)^{\frac{1}{n}}\left(c+\frac{1}{3}\right)^{\frac{1}{n}}\int_z^1\sum_0^\infty(-1)^p\frac{\frac{1}{n}(\frac{1}{n}-1)......(\frac{1}{n}-p+1)z^p dz}{1.2.3.4..............p}$$

$$=(R_n rn)^{\frac{1}{n}}\left(c+\frac{1}{3}\right)^{\frac{1}{n}}\sum_0^\infty(-1)^p\frac{\frac{1}{n}(\frac{1}{n}-1)......(\frac{1}{n}-p+1)(1-z^{p+1})}{1.2.3.4..............p(p+1)} \quad (40)$$

$v_r(r_1,z)$ is given by replacing r by r_1 *in* (40). For p = 0, the first term of the summation is equal to 1.

Similarly in the light of relation (30),

$$v_z(r,z)=(nR_n)^{1/n}r^{\frac{1-n}{n}}\frac{1+n}{n}\left(c+\frac{1}{3}\right)^{\frac{1}{n}}\sum_0^\infty(-1)^p\frac{\frac{1}{n}(\frac{1}{n}-1)......(\frac{1}{n}-p+1)p'}{1.2.3.....p(p+1)(p+2)} \quad (41)$$

where $p'=z^{(p+2)}-z(p+2)+p+1$

But for $0\leq r<r_1,$

$$v_z(r,0)=(nR_n)^{1/n}r^{\frac{1-n}{n}}\frac{1+n}{n}\left(c+\frac{1}{3}\right)^{\frac{1}{n}}\sum_0^\infty(-1)^p\frac{\frac{1}{n}(\frac{1}{n}-1)......(\frac{1}{n}-p+1)(p+1)}{1.2.3.....p(p+1)(p+2)} \quad (42)$$

For p=0, the first term of the summation is 1.Needless to mention that p is an integer.

AXIAL FORCE AND LEAKAGE

The axial force[1] ie load capacity is re-determined in the context of the present design:

$$L=\pi r_1^2(p_1-_P)+2\pi\int_{r_1}^{r_2}(p-P)rdr$$

which in non-dimensional form in consequence of (10),(16) and (17) is given by dropping π:

$$\bar{L}=r_1^2+2\int_{r_1}^1 prdr=(r_1^2+1)/2 \quad (43)$$

(Dropping the bar sign on the right; $r_2=1, P=0,\ \bar{L}$=L/π)

In consequence of(3.1),(10) and (24), the dimensionless flux ie, leakage *after dropping the bar sign is given by*

$$Q=Lt_{\epsilon\to 0}\int_\epsilon^1 r_1 v_{r_1}dz=Lt_{\epsilon\to 0}r_1 v_{r_1}z|_\epsilon^1-\lim_{\epsilon\to 0}\int_\epsilon^1 r_1\frac{\partial v_{r_1}}{\partial z}zdz$$

$$Q=(R_n n)^{\frac{1}{n}}r_1^{\frac{n+1}{n}}\int_0^1\left[\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}\right]^{\frac{1}{n}}zdz \quad (44)$$

$(r_1\leq r\leq r_2)$;by use of (19).

Hence the leakage in case of Newtonian flow ie when n=1 is obtained as

$$Q_1=R_n r_1^2\int_0^1\left[\left\{c(1-z)+\left(\frac{1-z^3}{3}\right)\right\}\right]^1 zdz$$

$$=R_n r_1^2(\frac{c}{6}+\frac{1}{10}) \quad (45)$$

The continuity of fluid flow with the principle of conservation of mass entails that the flux through the seal ie leakage equals the rate Q′at which the fluid flows into the annular space through the ring. This aspect is ratified as follows by use of equation (27),(34) and (44) in non-dimensional form:

$$Q'=\int_0^{r_1}rv_z(r,0)dr=\int_0^{r_1}v_1 r^{\frac{1-n}{n}}rdr==(R_n n)^{\frac{1}{n}}r_1^{\frac{n+1}{n}}\int_0^1\left[\left\{c(1-z)+\right.\right.$$

1−z331nzdz=Q

The dimensionless leakage $Q_{1.5}$ for dilatant fluid with n=1.5 is given by

$$=(\frac{3}{2}R_{1.5})^{2/3}r_1^{5/3}\left(c+\frac{1}{3}\right)^{2/3}\left(\int_0^1\{1-\frac{cz+\frac{z^3}{3}}{(c+\frac{1}{3})}\right\}zdz \quad \text{For } z$$

$\leq 1,$ *expanding the integrand binomially and neglecting* the cube and other higher powers of $\frac{cz+\frac{z^3}{3}}{(c+\frac{1}{3})}$ which are very small in this context,

$$Q_{1.5}=(\frac{3}{2}R_{1.5})^{2/3}r_1^{5/3}\left(c+\frac{1}{3}\right)^{2/3}\int_0^1 z[1-\frac{2}{3}\frac{cz+\frac{z^3}{3}}{(c+\frac{1}{3})}+\frac{\frac{2}{3}(\frac{2}{3}-1)}{2\left(c+\frac{1}{3}\right)^2}(cz+\frac{z^3}{3})^2]dz$$

$= \left(\frac{3}{2}R_{1.5}\right)^{\frac{2}{3}} r_1^{\frac{5}{3}} \left(c+\frac{1}{3}\right)^{\frac{2}{3}} [\frac{1}{2} - \frac{2(5c+1)}{15(3c+1)} - \frac{1}{(3c+1)^2}(\frac{c^2}{4}+\frac{c}{9}+\frac{1}{72})]$ (46)

The dimensionless leakage for a pseudoplastic fluid(n<1), say, n=.5 is

$Q_{\frac{1}{2}} = (\frac{R_{.5}}{2})^2 r_1^3 \int_0^1 \left\{c(1-z) + \left(\frac{1-z^3}{3}\right)\right\}^2 zdz$ (Integrating by parts with second form as $\int zdz$)

$= \left(\frac{R_{.5}}{2}\right)^2 r_1^3 \int_0^1 \left\{c(1-z) + \left(\frac{1-z^3}{3}\right)\right\}^2 (c+z^2)\, z^2 dz$

$= \left(\frac{R_{.5}}{2}\right)^2 r_1^3 (\frac{c^2}{12}+\frac{4c}{45}+\frac{1}{40})$ (47)

The dimensionless leakage for a dilatant fluid, n=2,is obtained as

$$Q_2 = (2R_2)^{1/2} r_1^{3/2} \int_0^1 \left\{c(1-z) + \left(\frac{1-z^3}{3}\right)\right\}^{1/2} zdz$$

Neglecting cube and other powers

of $\frac{cz+\frac{z^3}{3}}{\left(c+\frac{1}{3}\right)} \leq 1\, for\, z \leq$

1 *in the binomial expansion of the above integrand, we get*

$Q_2 = (2R_2)^{1/2} r_1^{3/2} \left(c+\frac{1}{3}\right) \int_0^1 z[1 - \frac{1}{2}\frac{cz+\frac{z^3}{3}}{\left(c+\frac{1}{3}\right)} + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2\left(c+\frac{1}{3}\right)^2}(cz+\frac{z^3}{3})^2]dz$

$= (2R_2)^{1/2} r_1^{3/2} \left(c+\frac{1}{3}\right) [\frac{1}{2} - \frac{c+\frac{1}{4}}{2(3c+1)} - \frac{1}{8}\frac{\frac{9c^2}{4}+c+\frac{1}{8}}{(3c+1)^2}]$ (48)

Finally we find the flux for pseudoplastic fluid with n=.75:

$Q_{.75} = \left(3\frac{R_{.75}}{4}\right)^{\frac{4}{3}} r_1^{\frac{7}{3}} \int_0^1 \left\{c(1-z) + \left(\frac{1-z^3}{3}\right)\right\}^{\frac{4}{3}} zdz = \left(3\frac{R_{.75}}{4}\right)^{\frac{4}{3}} r_1^{\frac{7}{3}} \left(c+\right.$
1343[12−4c3+4153c+1+29c+132(c24+c9+172)]
(49)

Numerical example 1

Utilizing the foregoing equations with the following data E_u=5,r_1=.9(dimensionless) R_n=20 for n=1,2,1.5,.5,.75 while applying the foregoing formulations and derivations, we find with desired approximation :c=.4205 and the dimensionless seal leakages for the aforesaid values of n:

$Q_1 = 2.7540$, $Q_2 = 1.3681$, $Q_{1.5} = 1.962$, $Q_{.5}$=5.6206 and $Q_{.75}$=3.6689

and the dimensionless axial force L=.9051 (50)

5. Discussion

The expressions for the fluid velocity components and for the leakage derived in the foregoing analysis reveal that they increase (1) with the increase in thickness(h) of the film,(2) with the increase in inner radius of the seal ring,(3) with the increase in Reynold Ratio number(n), (4) with the decrease in Euler number(E_u) and (5) with the decrease in index number(n), while the other parameters are kept constant in each case. Case(5) is confirmed with the above numerical example. The variation of dimensionless leakage with the variation of index number of the fluid is more or less in agreement with the relevant results[1] and as such reaffirms that the flux i.e. leakage is the least for dilatants fluids(n>1) and is the highest for pseudoplastic fluids(n<1).The non- dimensional fluid pressure steadily decreases from its value at the inlet to zero at the outlet of the seal, obeying, in consequence of (17.1) and (17), equation(17.2).

The term contributing the centrifugal force due to rotation of the seal plate, in the expressions for velocity components in r- and z- directions is the factor $\frac{1+z+z^2}{3}$, which can be regarded as negligibly small provided the value of c is sufficiently large compared to the maximum value of this factor ie c>>1.Since the value of c given by (17.1) increases with the decrease in Euler's number E_u , with the desired accuracy the centrifugal force becomes very small for low values of E_u and therefore can be neglected. However, neglecting the centrifugal force the foregoing two fluid-velocity components can be given by integrals(24) and (20):

$v_{r0(r,z)} = (cnR_n)^{1/n} \int_z^1 (1-z)^{1/n} dz = (cnR_n)^{1/n} \frac{n(1-z)^{\frac{n+1}{n}}}{n+1}$ (51)

$v_{z0} = (cnrR_n)^{1/n}(1+n)(1-z)^{\frac{2n+1}{n}} \frac{1}{r}(\frac{1}{n+1} - \frac{1}{2n+1})$

$v_{z0} = (cnrR_n)^{1/n} (1-z)^{\frac{2n+1}{n}} \frac{n}{r(2n+1)}$ (52)

Similarly the leakage neglecting the centrifugal force can be obtained by use of integral (44):

$Q_0 = (cnr_1 R_n)^{1/n} r_1 \int_0^1 z(1-z)^{1/n} dz$ (by use of properties of definite integral)

$Q_0 = (cnr_1 R_n)^{1/n} r_1 \int_0^1 (1-z) z^{1/n} dz$

$Q_0 = \frac{(cnr_1 R_n)^{1/n} r_1 n^2}{(n+1)(2n+1)}$ (53)

and consequently the dimension leakages without 'centrifugal force' with reference to the preceding numerical example give

$(Q_0)_1 = 1.1340$, $(Q_0)_2$=.9346, $(Q_0)_{1.5}$=1.0903, $(Q_0)_{.5}$=1.0828, $(Q_0)_{.75}$=1.1690 (54)

which reckon almost the same deviations from the values with inclusion of the centrifugal force as cited in earlier work[1].

Reference

[1]. Prawal Sinha, C.M. Rodkiewcz and J.S. Kennedy, Centrifugal effects in radial face seal with power-law fluid, *Indian journal of Pure and Applied Mathematics*, 1993; 24(3): 209-220.

